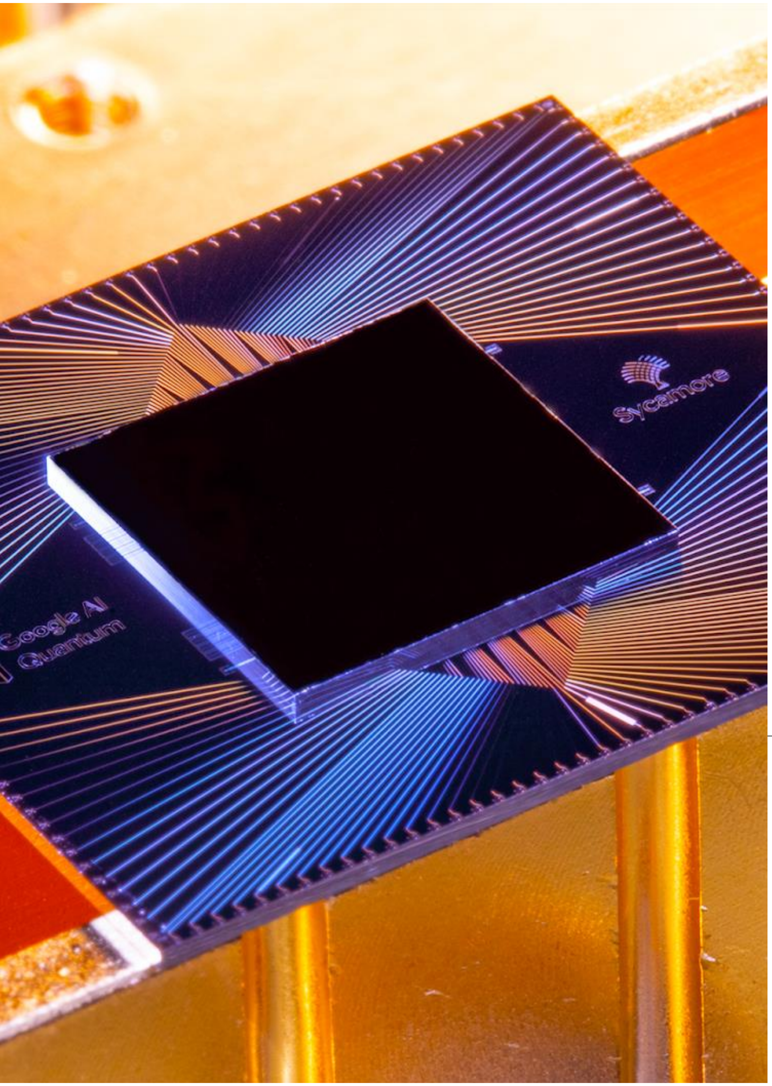


Classical Simulation of Quantum Computing

Quantum computing fundamentals

QUBITS, SUPERPOSITION AND ENTANGLEMENT

Qubits

A quantum computer manipulates **qubits**. A (classical) bit can be either 0 or 1, a qubit can be 0, 1 or anything "in between". 0 and 1 are **vectors** in a **2-dimensional Hilbert Space**.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

A qubit can be in any linear combination of 0 and 1, called a **superposition**.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \text{with} \quad |\alpha|^2 + |\beta|^2 = 1$$

Quantum superposition

An example of states in superposition are the $|+\rangle$ and $|-\rangle$ states, both of which are a symmetrical combination of $|0\rangle$ and $|1\rangle$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Superposition is the key for **quantum parallelism**: if f is a linear transformation, starting from a superposition of inputs we can get a superposition of outputs with just one application of f

$$f(|+\rangle) = \frac{1}{\sqrt{2}}f(|0\rangle) + \frac{1}{\sqrt{2}}f(|1\rangle)$$

Entanglement

A system of *n* qubits can be described as a $\mathbf{2^n}$**-dimensional Hilbert space**

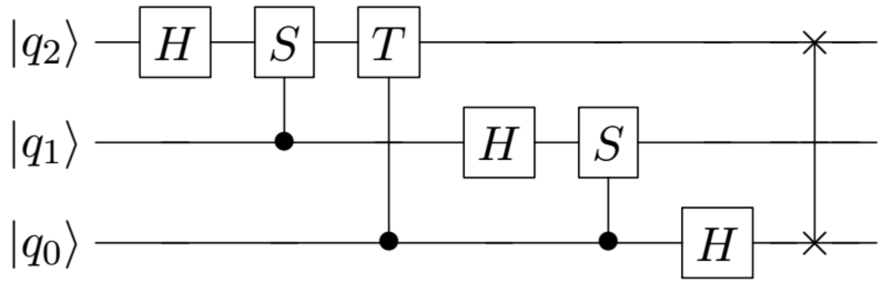
$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Some states are **separable**, they can be decomposed in two separate subsystems. The others are called **entangled states**.

$$\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = |++\rangle \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = |\Phi^+\rangle$$

Quantum circuits

A quantum program is implemented as a **circuit**, composed of *n* wires and *m* **gates**



Gates are linear transformation on a 2^n-dimensional Hilbert space, formalized as **matrices**

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \qquad CS = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & i\end{pmatrix}$$

Measurements

What happens if we **measure** a state in superposition? It **decays** into one of the classical states. When we measure $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ it gives a 0 outcome with probability $|\alpha|^2$ and a 1 outcome with probability $|\beta|^2$.

If we measure the finale state $|\psi\rangle = C|0..0\rangle$ of a circuit C with input $|0..0\rangle$, we get as an outcome a **bitstring** s $\in \{0, 1\}^n$, and the probability of each possible s depends on the transformations applied by C. In other words, each circuit defines a (discrete) **probabilistic distribution** of bitstrings.

Quantum supremacy

IS *SIMULATING* LARGE-SCALE QUANTUM SYSTEMS MERELY REALLY, REALLY HARD, OR IS IT RIDICULOUSLY HARD?

Quantum Supremacy

Quantum supremacy is the capability of quantum computers to solve a problem that no classical computer can solve in a feasible amount of time

Quantum Supremacy has not been reached yet!

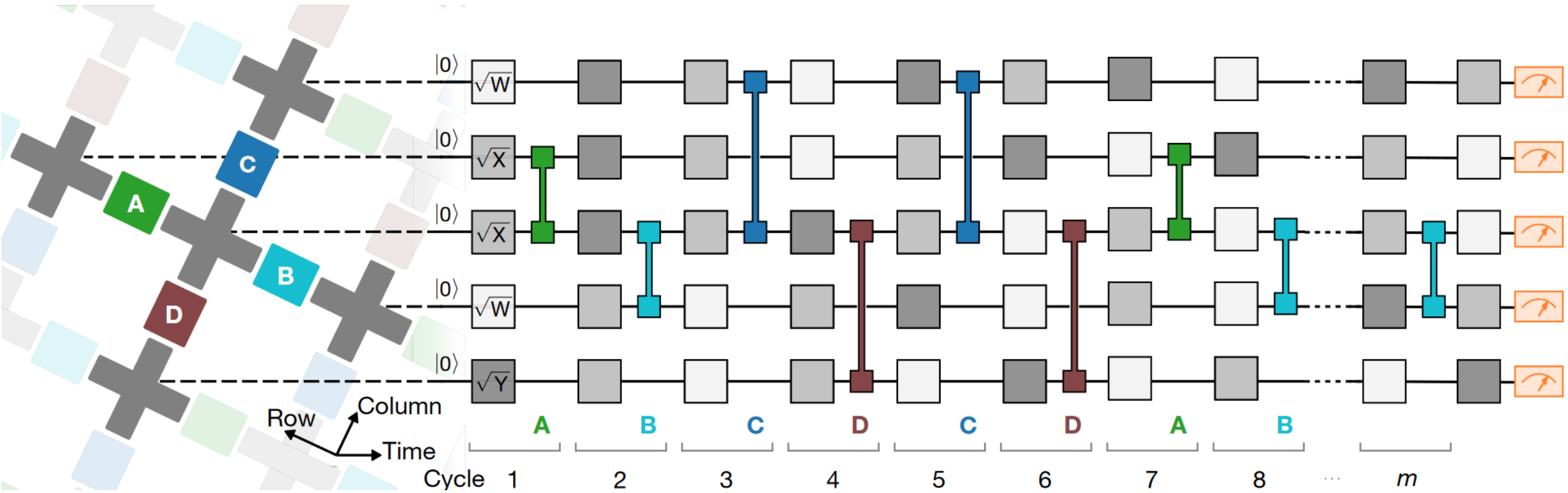
Which problem? **Any problem**, no matter how useless it might be.

A recently studied problem is a simulation problem: *"Given a random quantum circuit **C**, with **n** qubits and **m** cycles, replicate the probabilistic distribution of outcomes of C".*

The Sycamore attempt

The quantum computer built by Google, **Sycamore**, features n = 53 qubits and m = 20 cycles.

Sycamore uses faulty qubits, so it yields an **approximation** of the desired distribution. It was predicted that simulating Sycamore with the same fidelity would take 10 thousand years!



Classical simulation

The simplest idea is the Shrödinger algorithm, calculating all the coefficients

$$|\psi\rangle = \alpha|0 \ldots 00\rangle + \beta|0 \ldots 01\rangle + \gamma|0 \ldots 10\rangle \ldots$$

For the Sycamore circuit, you need to compute 2^{52} complex values!

There are a lot of improvements: Stabilizer decomposition, Shrödinger-Feynman algorithm, Tensor network contractions..

Implementing the **tensor network** approach on an exaFLOP computer would reach the same fidelity of Sycamore in shorter time

Let's discuss!

What we have seen:

- Quantum computers allow for "simultaneous" parallel computations
- Simulating quantum computers is (exponentially) difficult
- Also building quantum computers is difficult!

Which will be the future:

- ❑ Place you bet, will we reach quantum supremacy?
- ❑ Will we reach quantum advantage for useful problems?
- ❑ What are some problems in your field that would benefit the most from quantum parallelism?

Thank you for your attention!